Analyzing Exponential Functions

These notes are intended as a summary of section 5.2 (p. 344 – 348) in your workbook. You should also read the section for more complete explanations and additional examples.

Exponential Functions
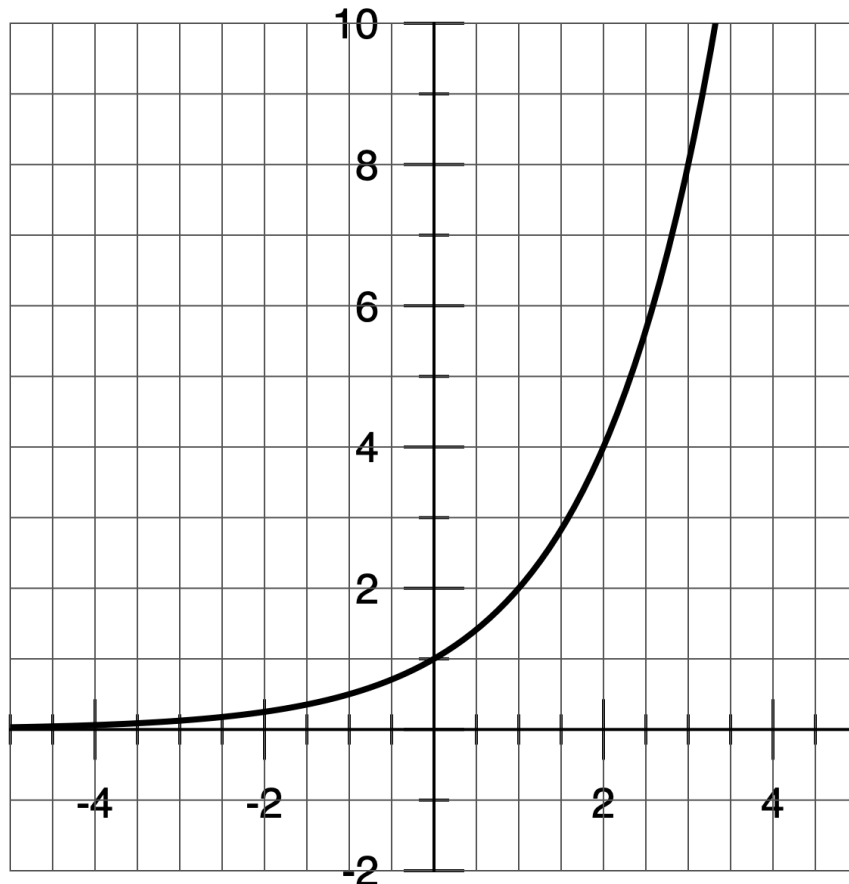
An **exponential function** is any function of the form

$$f(x) = a^x \qquad \begin{cases} a \neq 1 \\ a > 0 \end{cases}$$
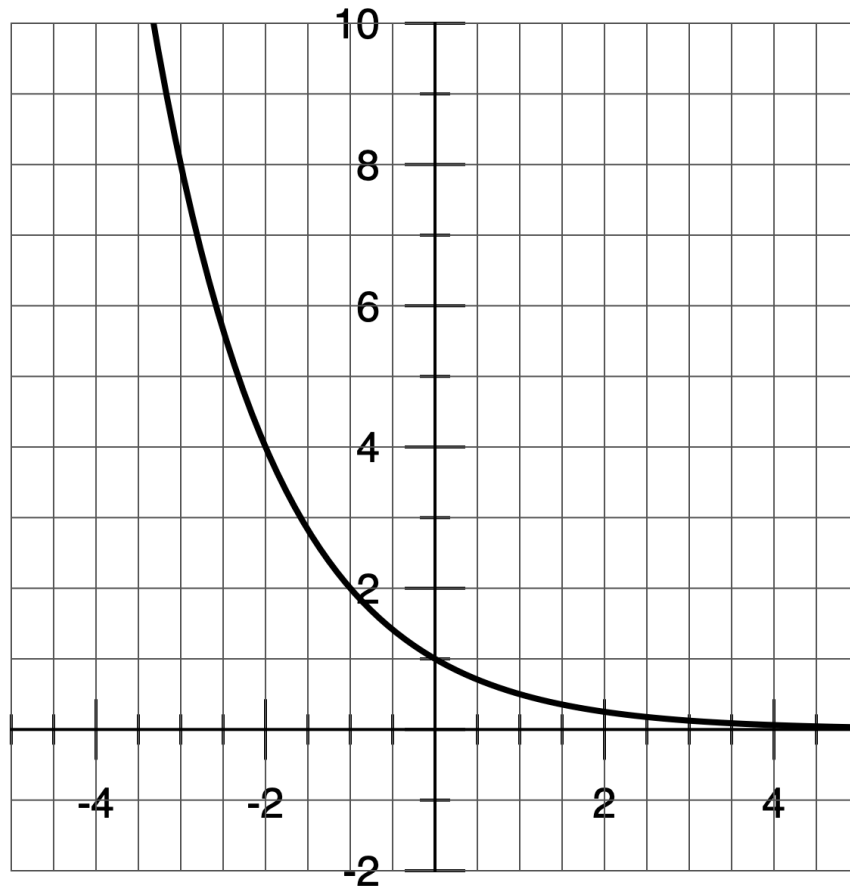
where a is the **base** and x is the **exponent**.

Graphing Exponential Functions

In the Math Lab, we observed the graphs of various exponential functions. In general, the graphs had one of two appearances.

1. The graph of $f(x) = a^x$ when $a > 1$.

2. The graph of $f(x) = a^x$ when $0 < a < 1$.

Note the following properties of the graph of $f(x) = a^x$:

1. When $a > 1$, y increases as x increases. The function is said to be **increasing**.

2. When $0 < a < 1$, y decreases as x increases. The function is said to be **decreasing**.

3. The y-intercept of the graph is always 1.

4. The point $(1, a)$ is always on the graph.

5. The x-axis $(y = 0)$ is a horizontal asymptote.

6. The graph has no x-intercepts.

7. The domain of the function is $x \in \mathbb{R}$.

8. The range of the function is $y > 0$.

Example 1 (sidebar p. 345)

a) Graph $y = \left(\dfrac{1}{3}\right)^x$.

x	y
−2	
−1	
0	
1	
2	
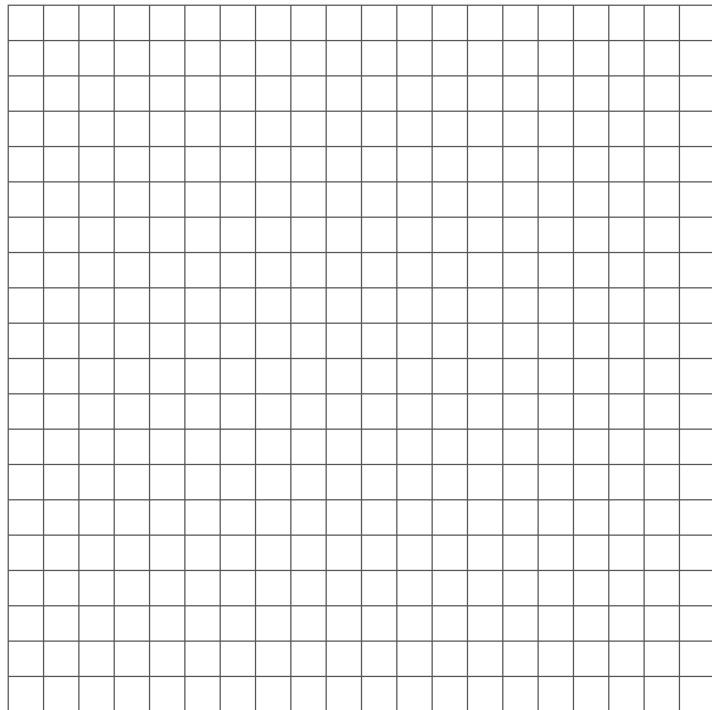
b) Determine:

    i)   the effect on y when x increases by 1

    ii)   whether the function is increasing or decreasing

    iii)   the intercepts

    iv)   the equations of any asymptotes

    v)   the domain of the function

    vi)   the range of the function

Transforming Exponential Functions

The image graph $y = ca^{b(x-h)} + k$ is the graph of $y = a^x$:

- stretched vertically by a factor of $|c|$

- stretched horizontally by a factor of $\dfrac{1}{|b|}$

- reflected in the x-axis when $c < 0$

- reflected in the y-axis when $b < 0$

- translated k units vertically

- translated h units horizontally

Note: Remember that transformations must be applied in the correct order (stretches, then reflections, then translations).
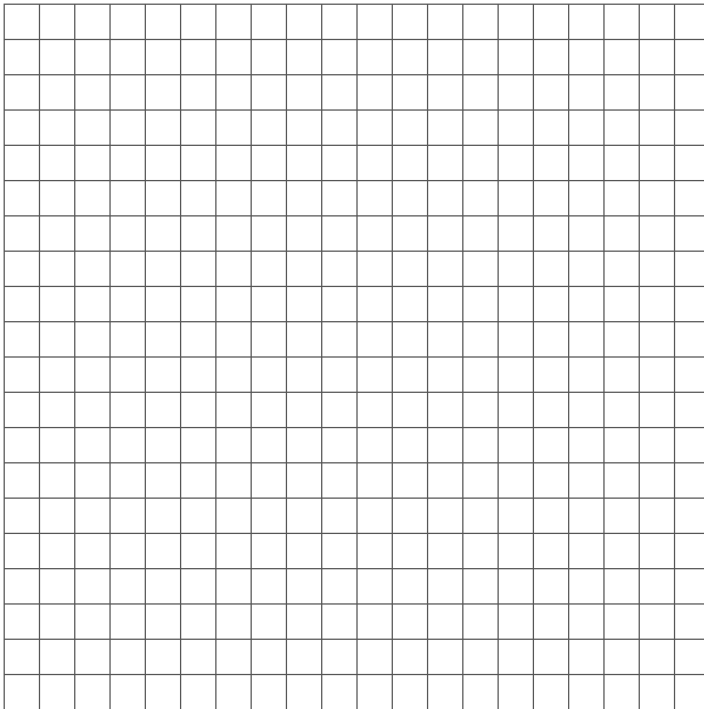
Example (not in workbook)

Graph $y = -5\left(3^{2(x+4)}\right) - 7$.
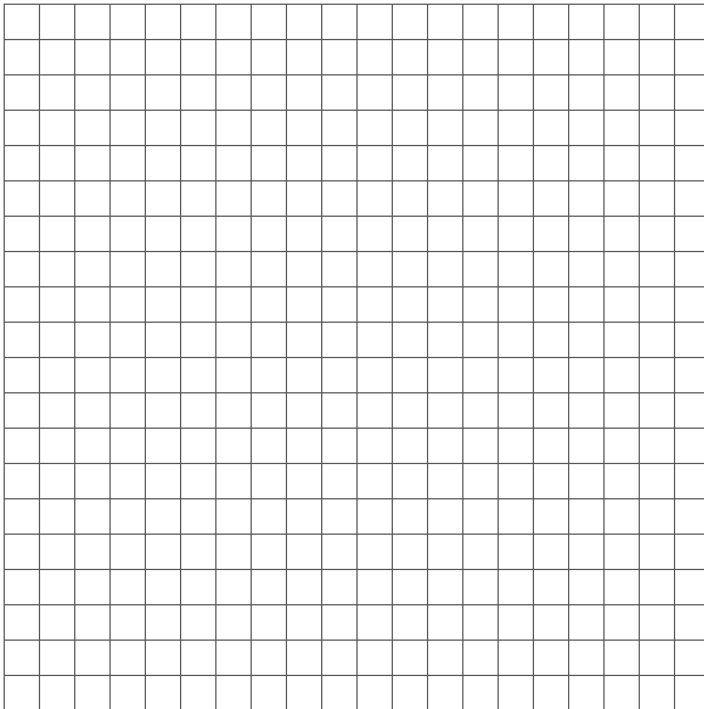
Example 2 (sidebar p. 347)

a) Use the graph of $y = 2^x$ to sketch the graph of $y = 3\left(2^{-x+2}\right)$.
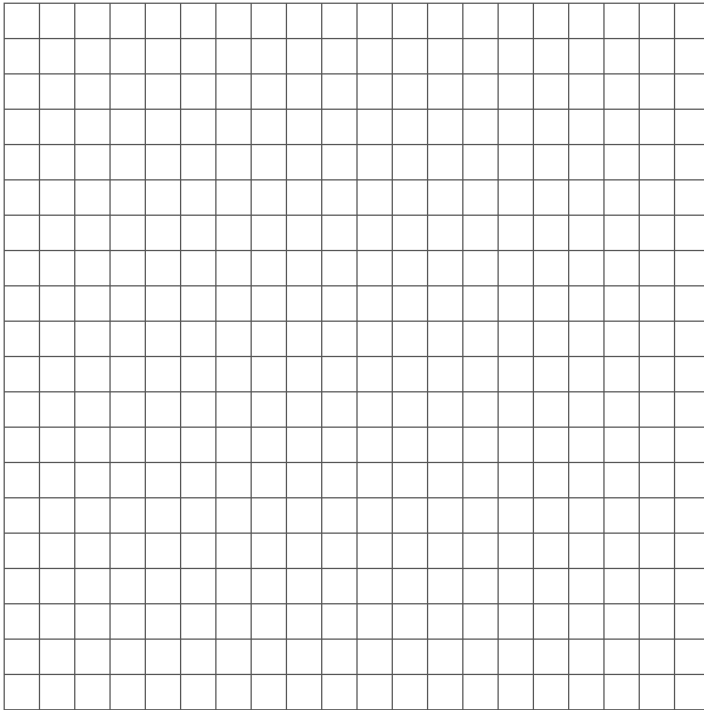

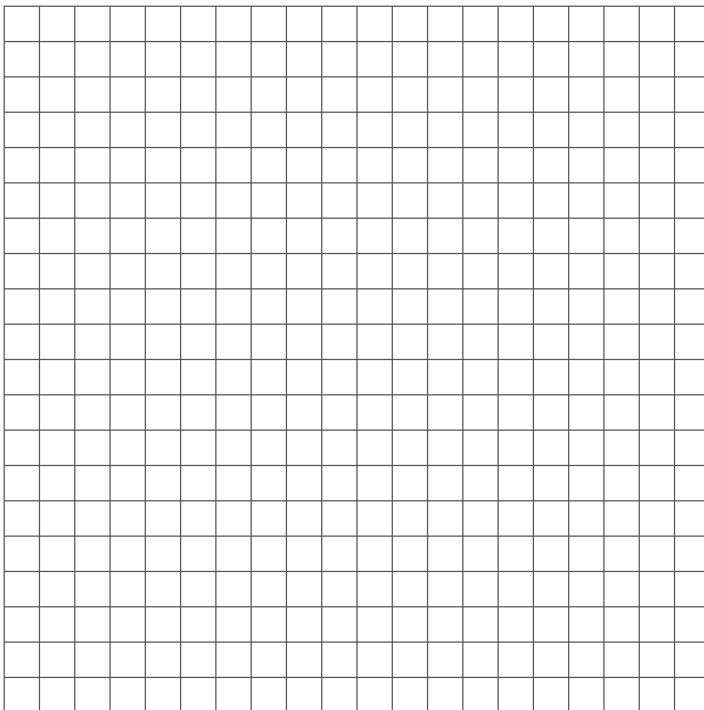b) From the graph of $y = 3\left(2^{-x+2}\right)$, determine:

    i)   whether the function is increasing or decreasing

    ii)   the intercepts

    iii)   the equation of the asymptote

    iv)   the domain of the function

    v)   the range of the function

Example (not in workbook)

a) Graph $y = e^x$.

b) Use the graph of $y = e^x$ to sketch the graph of $y = 3e^{x+2} - 5$.

Homework: #3, 5, 7, 10, 13 in the exercises (p. 349 – 355). Answers on p. 356.